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# Correspondence

## SIC-Based Detection With List and Lattice Reduction for MIMO Channels

Jinho Choi and Huan Nguyen

**Abstract**—To derive low-complexity multiple-input–multiple-output (MIMO) detectors, we combine two complementary approaches, i.e., lattice reduction (LR) and list within the framework of the successive interference cancellation (SIC)-based detection. It is shown that the performance of the proposed detector, which is called the SIC-based detector with list and LR, can approach that of the maximum-likelihood (ML) detector with a short list length. For example, the signal-to-noise ratio (SNR) loss of the proposed detector, compared with that of the ML detector, is less than 1 dB for a  $4 \times 4$  MIMO system with 16-state quadrature amplitude modulation (QAM) at a bit error rate (BER) of  $10^{-3}$  with a list length of 8.

**Index Terms**—Lattice reduction (LR)-based detection, list detection, successive interference cancellation (SIC), multiple-input–multiple-output (MIMO) detection.

### I. INTRODUCTION

In wireless communications, it is well known that the channel capacity can linearly increase with the number of antennas (provided that the numbers of transmit and receive antennas are the same) [1], [2]. Thus, to increase the channel capacity, the transmitter and receiver can be equipped with multiple antennas, and the resulting channel becomes a multiple-input–multiple-output (MIMO) channel. Various space-time architectures for signal transmission over MIMO channels are proposed to effectively exploit spatial and temporal diversity gain in [3] and [4].

In general, since more symbols are transmitted in MIMO systems, the detection complexity can be high. For example, the complexity of maximum-likelihood (ML) detection exponentially increases with the number of transmit antennas. Thus, various approaches are devised to reduce the complexity. The successive interference cancellation (SIC) approach is employed in [4]. The relation between SIC-based MIMO detection and the decision feedback equalizer (DFE) is exploited in [5]. In [6], the partial maximum *a posteriori* probability (MAP) principle is derived to discuss the optimality of SIC-based detection. List detectors are also considered for MIMO detection to obtain a soft decision in [7] and [8] based on [9].

In [10], a lattice reduction (LR)-based MIMO detector used as a low-complexity MIMO detector is first discussed. In [11], more LR-based MIMO detectors are proposed. It is shown that the performance of LR-based MIMO detectors using minimum square error (MMSE)-SIC approaches ML performance. An overview of LR-based detection can be found in [12]. In [13] and [14], it is shown that LR-based detection can achieve full diversity. This is an important observation as most low-complexity suboptimal MIMO detectors could

not exploit full diversity. It is noteworthy that a soft decision can also be obtained from the LR-based detection [15].

Although the Lenstra–Lenstra–Lovasz (LLL) algorithm, which is one of the LR algorithms, has a polynomial (average) complexity (for a certain class of random channel matrices) [16], [17], the complexity increases relatively rapidly with the number of basis vectors (or the number of transmit antennas). Thus, for a large MIMO system, the computational complexity of the LR-based detection would still be high. To further reduce the complexity, we can decompose a large MIMO detection problem into multiple small MIMO subdetection problems with SIC, as in [6]. Due to SIC, this approach would suffer from error propagation. To mitigate error propagation, the list detection approach can be adopted. The resulting detector has low complexity as the number of basis vectors in the subdetection problem is small. Due to list detection, the proposed detector can enjoy the tradeoff between complexity and performance, i.e., it has better mitigation against error propagation as the list length increases at the expense of increasing complexity.

### II. SYSTEM MODEL

Suppose that there are  $K$  transmit antennas and  $N$  receive antennas. The  $N \times 1$  received signal vector  $\mathbf{r}$  is given by

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n} \quad (1)$$

where  $\mathbf{H}$ ,  $\mathbf{s}$ , and  $\mathbf{n}$  are the  $N \times K$  channel matrix,  $K \times 1$  transmitted signal vector, and  $N \times 1$  noise vector, respectively. We assume that  $\mathbf{n}$  is a zero-mean circular complex Gaussian random vector with  $E[\mathbf{n}\mathbf{n}^H] = N_0\mathbf{I}$ . Let  $\mathcal{S}$  denote the signal alphabet for symbols, i.e.,  $s_k \in \mathcal{S}$ , where  $s_k$  is the  $k$ th element of  $\mathbf{s}$ , and its size is denoted by  $M$ , i.e.,  $M = |\mathcal{S}|$ .

We assume that  $N \geq K$  and consider the QR factorization of the channel matrix as  $\mathbf{H} = \mathbf{Q}\mathbf{R}$ , where  $\mathbf{Q}$  is unitary, and  $\mathbf{R}$  is upper triangular. We have

$$\mathbf{x} = \mathbf{Q}^H \mathbf{r} = \mathbf{R}\mathbf{s} + \mathbf{Q}^H \mathbf{n}. \quad (2)$$

Since the statistical properties of  $\mathbf{Q}^H \mathbf{n}$  are identical to those of  $\mathbf{n}$ ,  $\mathbf{Q}^H \mathbf{n}$  will be denoted by  $\mathbf{n}$ . If  $N = K$ , there are no zero rows in  $\mathbf{R}$ ; otherwise, the last  $N - K$  rows become zero. Thus, the last  $N - K$  elements of  $\mathbf{x}$  would be ignored for the detection if  $N > K$ . If there is no risk of confusion, hereinafter, we assume that the sizes of  $\mathbf{x}$ ,  $\mathbf{R}$ , and  $\mathbf{n}$  are  $K \times 1$ ,  $K \times K$ , and  $K \times 1$ , respectively.

### III. SIC-LIST-LR BASED DETECTION

The LR-based detectors in [10] and [11] have near-ML performance with relatively low complexity. It is shown that those LR-based detectors can achieve full diversity gain, just like the ML detector in [13] and [14]. Unfortunately, however, the complexity of LR can rapidly increase with the number of basis vectors, which implies that the complexity of the LR-based detectors may not be reasonably low for a large MIMO system. To avoid this problem, in this section, we propose an SIC-list-LR-based detection method within the framework of the partial MAP detection in [6]. The main idea of this method is to break a high-dimensional MIMO detection problem into multiple lower dimensional MIMO subdetection problems so that the complexity

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94 associated with LR can be reduced. The notion of the partial MAP  
 95 detection [6] is applied to include multiple lower dimensional MIMO  
 96 subdetection problems, together with the list detection approach.  
 97 To perform the proposed LR and list-based detection, we consider  
 98 the partition of  $\mathbf{x}$  as follows:

$$\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_3 \\ \mathbf{0} & \mathbf{R}_2 \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \end{bmatrix} \quad (3)$$

99 where  $\mathbf{x}_i$ ,  $\mathbf{s}_i$ , and  $\mathbf{n}_i$  are the  $K_i \times 1$   $i$ th subvectors of  $\mathbf{x}$ ,  $\mathbf{s}$ , and  $\mathbf{n}$ ,  $i =$   
 100 1, 2, respectively. Note that  $K_1 + K_2 = K$ . From (3), we can have two  
 101 lower dimensional MIMO subdetection problems to detect  $\mathbf{s}_1$  and  $\mathbf{s}_2$ .  
 102 It is straightforward to extend the partition into more than two groups.  
 103 However, for the sake of simplicity, we only consider the partition into  
 104 two groups, as in (3).

#### 105 A. Algorithm Description

106 In the proposed SIC-list-LR-based detection, the subdetection of  $\mathbf{s}_2$   
 107 is carried out first using the LR-based detector. Then, a list of candidate  
 108 vectors of  $\mathbf{s}_2$  is generated. With the list of  $\mathbf{s}_2$ , the subdetection of  $\mathbf{s}_1$  is  
 109 performed with the LR-based detector. The candidate vector in the list  
 110 is used for the SIC to mitigate the interference from  $\mathbf{s}_2$ . The proposed  
 111 SIC-list-LR-based detection is summarized here.

112 S1) The LR-based detection of  $\mathbf{s}_2$  is performed with the received  
 113 signal  $\mathbf{x}_2$ , i.e.,

$$\tilde{\mathbf{c}}_2 = \text{LRDet}(\mathbf{x}_2) \quad (4)$$

114 where  $\text{LRDet}$  is the function of the LR detection operation,  
 115 which will be discussed in Section III-B, and  $\tilde{\mathbf{c}}_2$  is the estimated  
 116 vector of  $\mathbf{s}_2$  in the corresponding LR domain. Note that there is  
 117 no interference from  $\mathbf{s}_1$  in detecting  $\mathbf{s}_2$ .

118 S2) A list of candidate vectors in the LR domain is generated by

$$\mathcal{C}_2 = \text{List}(\tilde{\mathbf{c}}_2) \quad (5)$$

119 where  $\text{List}$  is a function that chooses the  $Q$  closest vectors to  
 120  $\tilde{\mathbf{c}}_2$  ( $1 \leq Q \leq M^{K_2}$ ) in the LR domain. We will discuss the list  
 121 generation in Section III-C.

122 S3) The list of candidates of  $\mathbf{s}_2$ , which is denoted by  $\mathcal{S}_2$ ,  
 123 can be converted from  $\mathcal{C}_2$ . For convenience, denote  $\mathcal{S}_2 =$   
 124  $\{\tilde{\mathbf{s}}_2^{(1)}, \tilde{\mathbf{s}}_2^{(2)}, \dots, \tilde{\mathbf{s}}_2^{(Q)}\}$ .

125 S4) Once  $\mathcal{S}_2$  is available, the LR-based detection of  $\mathbf{s}_1$  can be carried  
 126 out with SIC, i.e.,

$$\mathbf{c}_1^{(q)} = \text{LRDet}(\mathbf{x}_1 - \mathbf{R}_3 \tilde{\mathbf{s}}_2^{(q)}) \quad (6)$$

127 where  $\tilde{\mathbf{s}}_2^{(q)}$  is the  $q$ th decision vector of  $\mathbf{s}_2$  from list  $\mathcal{S}_2$ .

128 S5) Let  $\tilde{\mathbf{s}}_1^{(q)}$  denote the signal vector corresponding to  $\mathbf{c}_1^{(q)}$  in the LR  
 129 domain and  $\tilde{\mathbf{s}}^{(q)} = [(\tilde{\mathbf{s}}_1^{(q)})^T \ (\tilde{\mathbf{s}}_2^{(q)})^T]^T$ ; the final decision of  $\mathbf{s}$   
 130 is found as

$$\tilde{\mathbf{s}} = \arg \min_{q=1,2,\dots,Q} \|\mathbf{x} - \mathbf{R} \tilde{\mathbf{s}}^{(q)}\|^2. \quad (7)$$

131 Note that a soft decision is also available from the list generated  
 132 in S5. There are  $Q$  candidate vectors for  $\mathbf{s}$ , and they can be used to  
 133 approximate the log-likelihood ratio as a soft decision, as in [18]. In the  
 134 succeeding sections, we will explain the proposed detection in detail.

#### 135 B. LR-Based Detection

136 In this section, we describe the LR-based detection used in steps S1)  
 137 and S4).

TABLE I  
SIGNALS AND PARAMETERS FOR THE LR-BASED  
DETECTION IN (4) AND (6)

Steps	$\mathbf{y}$	$\mathbf{A}$	$\mathbf{z}$	$\tilde{\mathbf{c}}$	$K_i$
S1)	$\mathbf{x}_2$	$\mathbf{R}_2$	$\mathbf{s}_2$	$\tilde{\mathbf{c}}_2$	$K_2$
S4)	$\mathbf{x}_1 - \mathbf{R}_2 \tilde{\mathbf{s}}_2^{(q)}$	$\mathbf{R}_1$	$\mathbf{s}_1$	$\tilde{\mathbf{c}}_1^{(q)}$	$K_1$

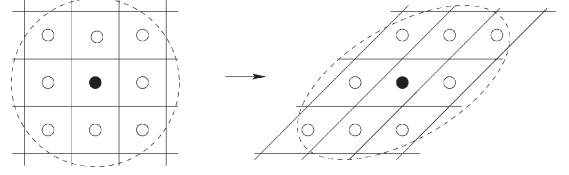


Fig. 1. List in different domains. (Left)  $\mathcal{C}_2$  in the LR domain, which is orthogonal and where the black dot represents  $\tilde{\mathbf{c}}_2$ . (Right)  $\mathcal{S}_2$  in the original domain, where the black dot represents  $\tilde{\mathbf{s}}_2$ .

Let  $\mathbb{C}$  denote the set of complex integers or Gaussian integers  $\mathbb{C} = \mathbb{Z} + j\mathbb{Z}$ , where  $\mathbb{Z}$  is the set of integers, and  $j = \sqrt{-1}$ . We assume  $\{ \alpha s + \beta | s \in \mathcal{S} \} \subseteq \mathbb{C}$ , where  $\alpha$  and  $\beta$  are the scaling and shifting coefficients, respectively. For example, for  $M$ -QAM, if  $M = 2^{2m}$ , we have

$$\mathcal{S} = \{ s = a + jb | a, b \in \{ \pm A, \pm 3A, \dots, \pm(2m-1)A \} \}$$

where  $A = \sqrt{(3E_s/2(M-1))}$ , and  $E_s = E[|s|^2]$  is the symbol energy. Thus,  $\alpha = 1/(2A)$ , and  $\beta = ((2m-1)/2)(1+j)$ . Note that the pair of  $\alpha$  and  $\beta$  is not uniquely decided.

Consider the MIMO detection with the following signal:

$$\mathbf{y} = \mathbf{A}\mathbf{z} + \mathbf{v} \quad (8)$$

where  $\mathbf{A}$  is a MIMO channel matrix,  $\mathbf{z} \in \mathcal{S}^{K_i}$  is the signal vector, and  $\mathbf{v}$  is a zero-mean Gaussian noise with  $E[\mathbf{v}\mathbf{v}^H] = N_0\mathbf{I}$ . We scale and shift  $\mathbf{y}$  as

$$\begin{aligned} \mathbf{d} &= \alpha\mathbf{y} + \beta\mathbf{A}\mathbf{1} \\ &= \mathbf{A}(\alpha\mathbf{z} + \beta\mathbf{1}) + \alpha\mathbf{v} \\ &= \mathbf{A}\mathbf{b} + \alpha\mathbf{v} \end{aligned} \quad (9)$$

where  $\mathbf{1} = [1 \ 1 \ \dots \ 1]^T$ , and  $\mathbf{b} = \alpha\mathbf{z} + \beta\mathbf{1} \in \mathbb{C}^{K_i}$ . Let

$$\tilde{\mathbf{A}} = \mathbf{A}\mathbf{U} \quad (10)$$

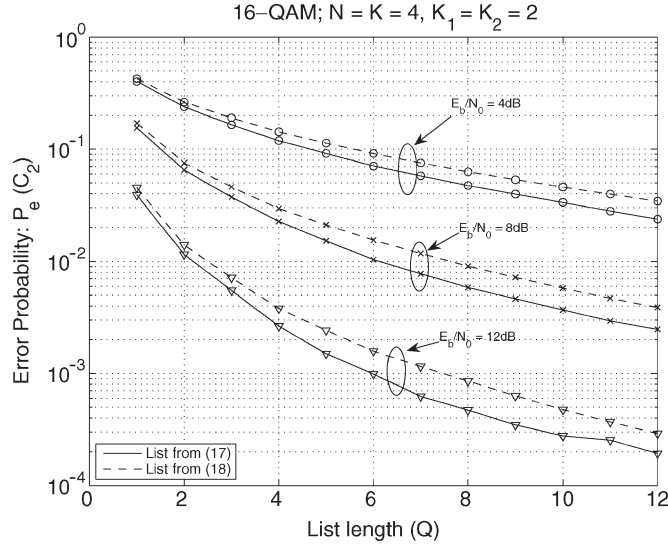
where  $\mathbf{U}$  is a unimodular matrix.<sup>1</sup> Using any LR algorithm, including the LLL algorithm [16], we can find the value of  $\mathbf{U}$  that makes the column vectors of  $\tilde{\mathbf{A}}$  shorter. It follows that

$$\begin{aligned} \mathbf{d} &= \mathbf{A}\mathbf{U}\mathbf{U}^{-1}\mathbf{b} + \alpha\mathbf{v} \\ &= \tilde{\mathbf{A}}\mathbf{c} + \alpha\mathbf{v} \end{aligned} \quad (11)$$

where  $\mathbf{c} = \mathbf{U}^{-1}\mathbf{b}$ . Note that, as the basis vectors are complex, we can use complex LR algorithms [19] or convert a complex matrix into a real matrix, as in [11]. The MMSE filter for estimating  $\mathbf{c}$  is given by

$$\begin{aligned} \mathbf{W}_{\text{MMSE}} &= \min_{\mathbf{W}} E \left[ \|\mathbf{W}^H(\mathbf{d} - \bar{\mathbf{d}}) - (\mathbf{c} - \bar{\mathbf{c}})\|^2 \right] \\ &= (\bar{\mathbf{A}}\text{cov}(\mathbf{c})\bar{\mathbf{A}}^H + |\alpha|^2 N_0 \mathbf{I})^{-1} \bar{\mathbf{A}}\text{cov}(\mathbf{c}) \\ &= (\mathbf{A}\mathbf{A}^H \alpha^2 E_s + |\alpha|^2 N_0 \mathbf{I})^{-1} \mathbf{A}\mathbf{U}^{-H} \alpha^2 E_s \end{aligned} \quad (12)$$

<sup>1</sup>A unimodular matrix is a square integer matrix with determinant  $\pm 1$ .

Fig. 2. Error probability with the list of  $\mathbf{c}_2$  for various list lengths.

157 where  $\bar{\mathbf{d}} = E[\mathbf{d}] = \beta \mathbf{A} \mathbf{1}$ ,  $\bar{\mathbf{c}} = E[\mathbf{c}] = \mathbf{U}^{-1} \beta \mathbf{1}$ , and  $\text{cov}(\mathbf{c}) =$   
 158  $|\alpha|^2 \mathbf{U}^{-1} \mathbf{U}^{-H} E_s$  since

$$\text{cov}(\mathbf{c}) = \text{cov}(\mathbf{U}^{-1} \mathbf{b}) = \alpha^2 E_s \mathbf{I}.$$

159 The estimate of  $\mathbf{c}$  is given by

$$\hat{\mathbf{c}} = \bar{\mathbf{c}} + \mathbf{W}_{\text{MMSE}}^H (\mathbf{d} - \bar{\mathbf{d}}). \quad (13)$$

160 In Table I, the signals and parameters for the LR-based MMSE de-  
 161 tecton for each step are shown. Note that other approaches, including  
 162 the LR-based MMSE-SIC detector in [11] or non-LR-based detectors,  
 163 can also be used for subdetection.

#### 164 C. List Generation in the LR Domain

165 To avoid or mitigate the error propagation, the use of a list of  
 166 candidate vectors of  $\mathbf{s}_2$  in detecting  $\mathbf{s}_1$  is crucial. Using the ML metric,  
 167 we can find the candidate vectors for the list  $\mathcal{S}_2$ . Let

$$f(\mathbf{r} | \hat{\mathbf{s}}_2^{(1)}) \geq f(\mathbf{r} | \hat{\mathbf{s}}_2^{(2)}) \geq \dots \geq f(\mathbf{r} | \hat{\mathbf{s}}_2^{(M^{K_2})})$$

168 where  $f(\mathbf{r} | \mathbf{s})$  is the likelihood function of  $\mathbf{s}$  for a given  $\mathbf{r}$ , and  $\hat{\mathbf{s}}_2^{(q)}$  is the  
 169 symbol vector that corresponds to the  $q$ th largest likelihood. With log-  
 170 likelihood values, we can also find the candidate vectors as follows:

$$\|\mathbf{r} - \mathbf{R}_2 \hat{\mathbf{s}}_2^{(1)}\|^2 \leq \|\mathbf{r} - \mathbf{R}_2 \hat{\mathbf{s}}_2^{(2)}\|^2 \leq \dots \leq \|\mathbf{r} - \mathbf{R}_2 \hat{\mathbf{s}}_2^{(M^{K_2})}\|^2.$$

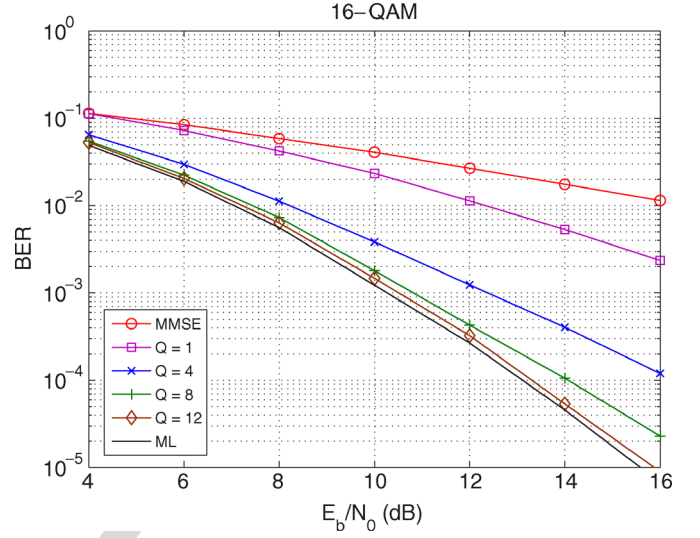
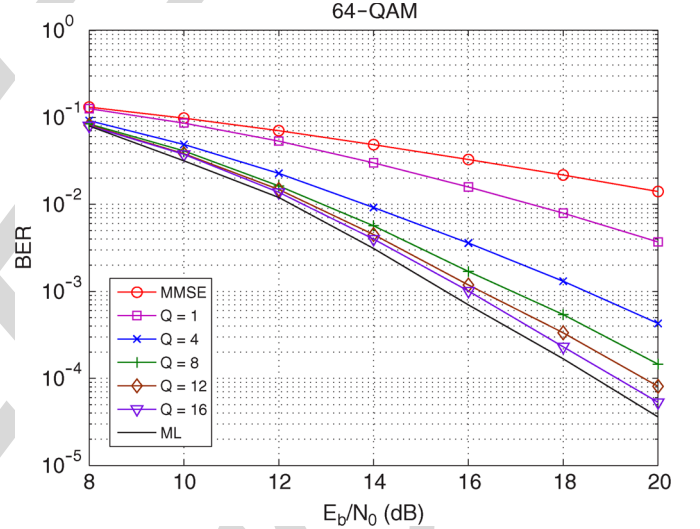
171 Therefore, the ML-based list becomes

$$\mathcal{S}_2 = \{\hat{\mathbf{s}}_2^{(1)}, \hat{\mathbf{s}}_2^{(2)}, \dots, \hat{\mathbf{s}}_2^{(Q)}\}. \quad (14)$$

172 However, for each log-likelihood value, we need to perform a  
 173 matrix-vector multiplication. Thus, the resulting computational com-  
 174 plexity could be high.

175 To avoid high computational complexity in generating the list, we  
 176 can find a suboptimal list in the LR domain that can be obtained with  
 177 a low complexity. Consider (9). According to Table I, let  $\mathbf{A} = \mathbf{R}_2$ ,  
 178  $\mathbf{d} = \alpha \mathbf{x}_2 + \beta \mathbf{A} \mathbf{1}$ , and  $\mathbf{b} = \alpha \mathbf{s}_2 + \beta \mathbf{1}$ . Then, from (10), we have

$$\|\mathbf{r} - \mathbf{R}_2 \mathbf{s}_2\| \propto \|\mathbf{d} - \mathbf{A} \mathbf{b}\| = \|\mathbf{d} - \bar{\mathbf{A}} \mathbf{c}\|. \quad (15)$$

Fig. 3. BER performance of a  $4 \times 4$  MIMO system with 16-QAM signaling.Fig. 4. BER performance of a  $4 \times 4$  MIMO system with 64-QAM signaling.

It is noteworthy that the metric on the right-hand side of (15) 179  
 is defined in the LR domain. Let  $\tilde{\mathbf{s}}_2$  be the signal vector in  $S^{K_2}$  180  
 corresponding to  $\tilde{\mathbf{c}}_2$ , and assume that  $\tilde{\mathbf{s}}_2$  is sufficiently close to  $\hat{\mathbf{s}}_2^{(1)}$ . 181  
 Then, we can have  $\mathbf{d} \simeq \bar{\mathbf{A}} \tilde{\mathbf{c}}_2$ . From this, the ML metric (ignoring a 182  
 scaling factor) for constructing the list in the LR domain becomes 183

$$\|\mathbf{d} - \bar{\mathbf{A}} \mathbf{c}\| \simeq \|\bar{\mathbf{A}} \tilde{\mathbf{c}}_2 - \bar{\mathbf{A}} \mathbf{c}\| = \|\tilde{\mathbf{c}}_2 - \mathbf{c}\|_{\bar{\mathbf{A}}^H \bar{\mathbf{A}}} \quad (16)$$

where  $\|\mathbf{x}\|_{\bar{\mathbf{A}}} = \sqrt{\mathbf{x}^H \bar{\mathbf{A}} \mathbf{x}}$  is a weighted norm. The list in the LR 184  
 domain becomes 185

$$\mathcal{C}_2 = \{\mathbf{c}_2 | \|\tilde{\mathbf{c}}_2 - \mathbf{c}\|_{\bar{\mathbf{A}}^H \bar{\mathbf{A}}} < r_{\bar{\mathbf{A}}}(Q)\} \quad (17)$$

where  $r_{\bar{\mathbf{A}}}(Q) > 0$  is the radius of an ellipsoid centered at  $\tilde{\mathbf{c}}_2$ , which 186  
 contains  $Q$  elements in the LR domain. If the column vectors of  $\bar{\mathbf{A}}$  187  
 or the basis vectors in the LR domain are orthogonal,  $\bar{\mathbf{A}}^H \bar{\mathbf{A}}$  becomes 188  
 diagonal. Furthermore, if they have the same norm,  $\bar{\mathbf{A}}^H \bar{\mathbf{A}} \propto \mathbf{I}$ . Thus, 189  
 for nearly orthogonal basis vectors of almost equal norm, the list of  $\mathbf{c}_2$  190  
 can be approximated as 191

$$\mathcal{C}_2 \simeq \tilde{\mathcal{C}}_2 = \{\mathbf{c}_2 | \|\tilde{\mathbf{c}}_2 - \mathbf{c}\| < r(Q)\} \quad (18)$$

TABLE II  
IMPACT OF THE MAXIMUM NUMBER OF COLUMN SWAPS IN THE LR ON BER (BER RAPIDLY DECREASES WITH THE NUMBER OF COLUMN SWAPS. ONLY A FEW NUMBER OF COLUMN SWAPS IS REQUIRED)

$E_b/N_0$	$N_{cs} = 1$	$N_{cs} = 2$	$N_{cs} = 3$	$N_{cs} = 4$
6 dB	$2.2472 \times 10^{-2}$	$2.2459 \times 10^{-2}$	$2.2459 \times 10^{-2}$	$2.2459 \times 10^{-2}$
10 dB	$1.9068 \times 10^{-3}$	$1.9062 \times 10^{-3}$	$1.9062 \times 10^{-3}$	$1.9062 \times 10^{-3}$

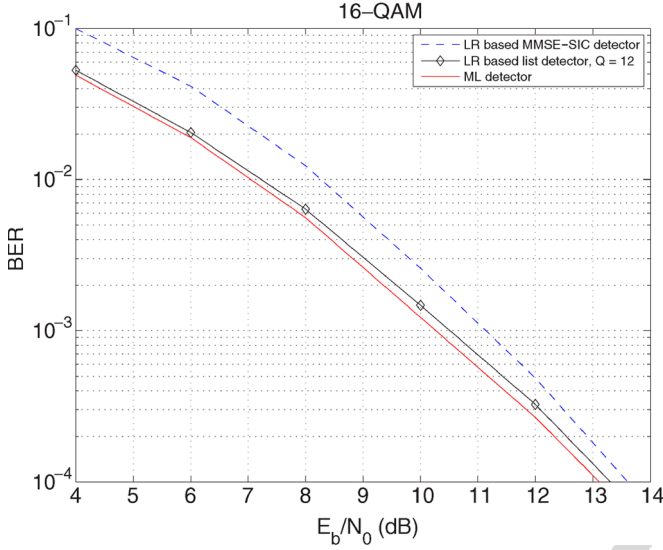


Fig. 5. BER performance comparison of a  $4 \times 4$  MIMO system with 16-QAM signaling.

192 where  $r(Q) > 0$  is the radius of a sphere centered at  $\tilde{\mathbf{c}}_2$ , which  
193 contains  $Q$  elements. Since the LR provides a set of nearly orthogonal  
194 basis vectors for the LR-based detection, we can see that the column  
195 vectors in  $\tilde{\mathbf{A}}$  in (10) are nearly orthogonal, as shown in Fig. 1, with  
196 a two-basis system. Let  $\tilde{\mathcal{S}}_2$  denote the list in the original domain  
197 obtained from  $\tilde{\mathcal{C}}_2$  as in step S3). Since no matrix-vector multiplications  
198 are required to generate  $\tilde{\mathcal{C}}_2$  or  $\tilde{\mathcal{S}}_2$ , we can use  $\tilde{\mathcal{S}}_2$  as the list in the  
199 proposed detector to reduce computational complexity.

200

#### IV. SIMULATION RESULT

201 In this section, we present simulation results. We mainly focus  
202 on the case of  $K = 4$ , particularly the case of  $K_1 = K_2 = 2$ . The  
203 elements of  $\mathbf{H}$  are independent zero-mean circular complex Gaussian  
204 random variables with unit variance. This case is particularly interest-  
205 ing as the Gaussian reduction, which can find the two shortest vectors  
206 in two-basis systems [10], [20], can be used for LR.

207 In the proposed SIC-list-LR-based detection, list length  $Q$  plays  
208 a key role in the tradeoff between complexity and performance. In  
209 general, it is desirable that the list has the true transmitted vector of  $\mathbf{c}_2$ .  
210 If not, the proposed detector will have an incorrect decision. If  $Q$  in-  
211 creases, the error probability that  $\mathcal{S}_2(\mathbf{C}_2)$  does not have the correct vec-  
212 tor of  $\mathbf{s}_2(\mathbf{c}_2)$ , which is denoted by  $P_e(\mathcal{S}_2)$  or  $P_e(\mathbf{C}_2)$ , decreases. Error  
213 probability  $P_e(\mathbf{C}_2)$  is considered for the MIMO system with 16-state  
214 quadrature amplitude modulation (16-QAM), and  $N = K = 4$ . Sim-  
215 ulation results are shown in Fig. 2, where the error probabilities are  
216 shown with two different lists in (17) and (18). As the list in (18)  
217 is suboptimal, the performance is worse. However, this performance  
218 degradation is not significant as the column vectors of  $\tilde{\mathbf{A}}$  are nearly  
219 orthogonal.

220 The bit error rate (BER) performance of a  $4 \times 4$  MIMO system  
221 with 16-QAM signaling is shown in Fig. 3. In this case, a near-ML  
222 performance can be achieved when  $Q \geq 8$ . For example, the signal-

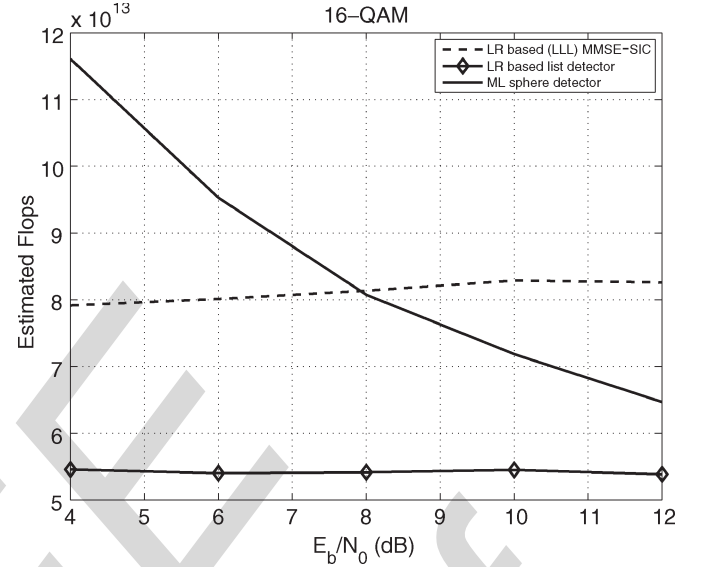


Fig. 6. Complexity comparison of a  $4 \times 4$  MIMO system with 16-QAM signaling.

to-noise ratio (SNR) loss of the proposed detector, compared with that  
223 of the ML detector, is less than 1 dB at a BER of  $10^{-3}$  when  $Q = 8$ . 224

Fig. 4 shows the simulation results with 64-state quadrature ampli-  
225 tude modulation (16-QAM). This result again confirms that the pro-  
226 posed SIC-list-LR-based detector can provide a near-ML performance  
227 with low complexity. At a BER of  $10^{-3}$ , the SNR loss is less than 1  
228 dB, compared with that of the ML detector when  $Q = 12$ . As the SNR  
229 or  $E_b/N_0$  increases, the SNR loss increases. However, by increasing  
230 list length  $Q$ , this loss can be reduced as the list length can exploit  
231 the tradeoff between performance and complexity. Note that a full  
232 diversity may not be achieved by the proposed detector with a fixed list  
233 length, as shown in Figs. 3 and 4. The relationship between diversity  
234 order and list length needs to be investigated in the near future. 235

In the LR-based detection, since the number of column swaps in  
236 the LR operation is not fixed, the complexity can vary from a channel  
237 matrix to another. Thus, in practice, the maximum number of column  
238 swaps can be fixed to limit the maximum complexity for two-basis  
239 systems. It is shown in [10] that the two shortest vectors can be found  
240 within two iterations for more than 99% of  $2 \times 2$  random matrices  
241 (of Rayleigh fading). However, when the number of column swaps  
242 is limited, the basis vectors may not be properly reduced for some  
243 channels, and the BER performance could be degraded because of it.  
244 To see the impact of the maximum number of column swaps, a  
245 simulation is considered with 16-QAM. Table II presents the BER  
246 performance when the maximum number of column swaps  $N_{cs}$  is  
247 limited. It is shown that the performance degradation is negligible,  
248 even though  $N_{cs} = 1$ . 249

For comparison purposes, we consider the BER performance of the  
250 LR-based MMSE-SIC detector, which is the best LR-based detector  
251 among the LR-based detectors proposed in [11]. The BER perfor-  
252 mance results are shown in Fig. 5. It is shown that the proposed  
253 detector can provide a performance that is better by about 1 dB than  
254 the LR-based MMSE-SIC detector at a BER of  $10^{-2}$ . Again, we 255



256 can confirm that the combination of the LR-based detection and list  
257 detection can improve the performance of the LR-based detector and  
258 is an effective means to approach the ML performance.

259 For complexity comparison, we can take the upper bound on  
260 the average number of LLL iterations in [17], which is given by  
261  $\bar{N}_{cs} = K^2 \log K / (N - K + 1)$ . (We ignore some minor terms to  
262 simplify the comparison.) For  $4 \times 4$  MIMO channels, we have  $\bar{N}_{cs} =$   
263  $K^2 \log K / (N - K + 1) = 16 \log 4$  for the LR-based MMSE-SIC  
264 detector and  $\bar{N}_{cs} = 2(K/2)^2 \log((K/2)/((N/2) - (K/2) + 1)) =$   
265  $8 \log 2$  for the proposed detector. This shows complexity reduction  
266 by more than half in terms of LLL iterations. Note that the proposed  
267 detector has additional complexity to build a list, which may offset the  
268 complexity advantage of the proposed detector over conventional LR-  
269 based detectors [11]. To further see the complexity of each detector,  
270 simulations are considered under the same environment, as shown in  
271 Fig. 5. Fig. 6 shows the estimated flops using MATLAB execution time  
272 that was obtained over all operations for each detector through simu-  
273 lations. The execution time is averaged over hundreds of thousands of  
274 channel realizations. The Sphere Schnorr–Euchner algorithm [21] is  
275 used for the ML decoding, whereas the LLL-reduced algorithm with  
276 reduction factor  $\delta = 3/4$  [16] is chosen for the LR-based MMSE-SIC  
277 detector [11]. (This is the same as that in Fig. 5.) No limitation on the  
278 number of iterations is imposed for any LR algorithm. The proposed  
279 LR-based list detector clearly requires the lowest execution time. We  
280 can also see that the execution time of the proposed detector is slightly  
281 higher than half of the execution time of the LR-based MMSE-SIC  
282 detector where the LLL-reduced algorithm is used.

## V. CONCLUDING REMARK

284 In this paper, we have derived an SIC-list-LR-based detector for  
285 MIMO detection using two complementary techniques, i.e., LR and  
286 list detection, within a framework of SIC-based detection. It was  
287 shown that the proposed detector has a near-ML performance with low  
288 complexity. The list length plays a key role in the tradeoff between  
289 performance and complexity. The performance is improved for a  
290 longer list length, whereas the complexity increases with list length  $Q$ .

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